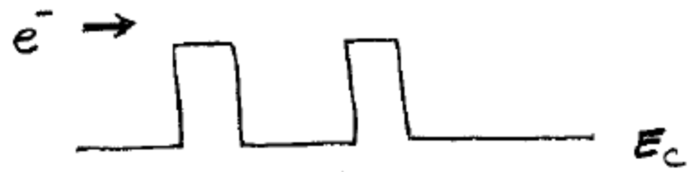
HW 4-5

Problem 1:

Resonant tunneling



Idealize as 2 δ-function potentials

Find:
(a) the transmission coefficient, T
(b) plot T vs. k (k is the wave vector of the incident electrons)

Problem 2:

Consider a Hall bar of length L_x and width L_y where we pass a current I_x and measure voltages V_x and V_y. Calculate V_x and V_y using the result (6.22) from the Drude model in a magnetic field.

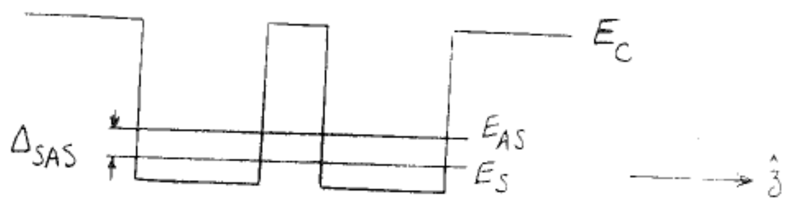
A two-dimensional electron gas is fabricated into a Hall bar with $L_x = 0.5$ mm and $L_y = 0.1$ mm. It gives $V_x = 0.13$ mV and $V_y = 0.31$ mV with $I_x = 1\,\mu$A and $B = 0.15$ T. Find the concentration and mobility of the 2DEG. Is the use of the low-field formulas justified?

Problem 3:

A sample of germanium shows no Hall effect. If the mobility of electrons in germanium is 3500 cm^2/V-sec and that of the holes is 1400 cm^2/V-sec, what fraction of the current in the sample is carried by electrons?

Problem 4:

Consider the following double-quantum-well (DQW) structure:



The electrons are free in the x-y plane but their motion in the z-direction is quantized. This quantization leads to the ground (symmetric) and excited (asymmetric) states which we denote by E_S and E_{AS} respectively, and define $\Delta_{SAS} = E_{AS} - E_S$.

Suppose we have two DQW samples, one with $\Delta_{SAS} = 1$ meV and one with $\Delta_{SAS} = 0.1$ meV. The *total* density of electrons in each DQW sample is $2\times10^{11}\text{cm}^{-2}$. Assume the QW's are made of GaAs ($m^* = 0.07m_e$), and ignore the higher (than E_{AS}) electric subbands, spin, or electron-electron interactions. Also, assume that in the following measurements we make electrical contact to both electron layers in parallel.

(a) Calculate the Fermi energy for each DQW sample.

(b) Can a low-temperature transport experiment tell the samples apart? Explain. Approximately how low should the temperature be?

(c) For each DQW sample, calculate the magnetic field (in Tesla) above which only one Landau level is occupied. (The magnetic field is applied in the z-direction.)

Problem 5:

1. AlAs has a six-fold degenerate conduction band minimum at the X-point of the Brillouin zone. The constant energy surfaces are ellipsoids of revolution characterized by two effective masses: longitudinal $m_l = 1.1m_0$ and transverse $m_t = 0.19m_0$ where $m_0 = 9.1 \times 10^{-31}$ kg is the free electron mass.

   It turns out that one can in fact realize a 2D electron system (2DES) in an AlAs quantum well, bounded by AlGaAs barriers, with the electrons occupying the x-valley ellipsoid(s). But it is not clear which of the ellipsoids are occupied since both stress (because of the lattice mismatch between AlAs and AlGaAs) and confinement can split the degeneracy of the ellipsoids.

   In figure 2 and figure 3 cyclotron resonance and magnetotransport data, taken with the magnetic field perpendicular to the surface of a 2DES in an AlAs quantum well grown on GaAs(100) substrate, are shown.

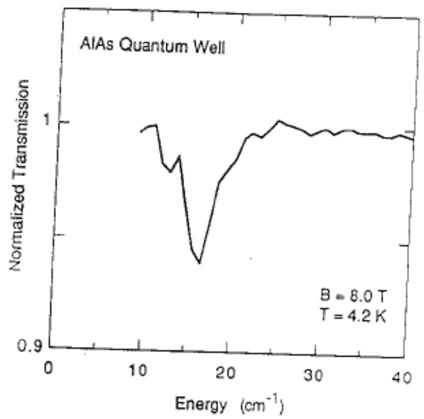


FIG. 2. Cyclotron resonance spectrum for the AlAs 2DES obtained by measuring the transmission of far-infrared radiation through the sample. T ission at $B=8$ T normalized to that at $B=0$ is shown as a func- the incident energy.

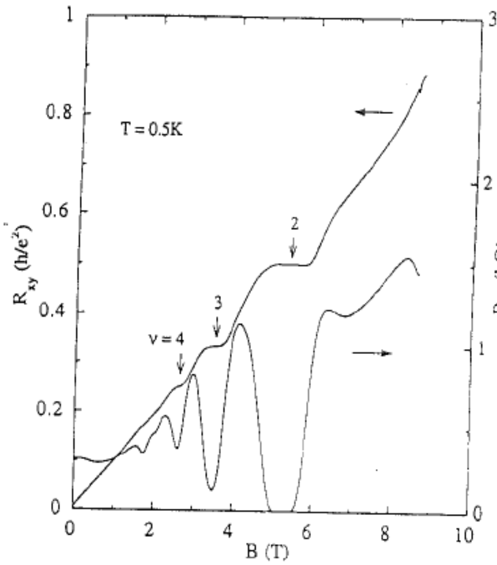


FIG. 3. The magnetotransport coefficients R_{xx} and R_{xy} vs magnetic field B at $T=0.5$ K. The vertical arrows indicate the Landau filling factors at which integral quantum Hall effect is observed.

From the data:

(1) Determine which ellipsoid(s), i.e., those with their major axes in the 2DES plane or perpendicular to the plane, are occupied. Explain.

(2) What is the areal density of the (2DES)? What is the Fermi energy?

(3) Is the data of figure 3 consistent with your answer to part (1)? Explain possible reason(s) for discrepancy.

1 cm^{-1} = 0.124 meV